

## Lesson 3 Mathematics around Us

### Reading 1

#### Prime Numbers: Not Just for Mathematicians

The numbers 2, 3, 5, 7, 11, 13, 53, and 71 all share a notable feature. Can you guess what it is? They are all prime numbers. A prime number is a number that is greater than one and can only be divided by one and itself. These special numbers have fascinated mathematicians for a very long time. More than two thousand years ago, the ancient Greek mathematician Euclid proved that there is an infinite number of them. However, no one has ever been able to discover a pattern to their appearance.

The unique and mysterious nature of prime numbers has also been a source of inspiration for writers, musicians, and artists. For example, Carl Sagan, who was both an astronomer and a writer, based his book *Contact* on the idea of using prime numbers to communicate with aliens. Moreover, the French composer Olivier Messiaen used prime numbers to create special and unpredictable rhythms in his music. Prime numbers aren't just important to mathematicians and artists, though. We might not realize it, but they influence many different aspects of the world around us, and we even make use of their unique nature in our daily lives.

#### Cicadas' Survival Secret

Certain cicadas found in North America, called periodical cicadas, provide a fantastic example of prime numbers' importance in nature. Periodical cicadas spend most of their life under the ground, but after a period of either 13 or 17 years, the whole huge population emerges. At this time, they appear for just a few weeks in order to reproduce before they die. Interestingly, both 13 and 17 are prime numbers, and some scientists have proposed that this fact might be crucial to the survival of periodical cicadas.

Because 13 and 17 are prime numbers, these cicadas can avoid predators that appear periodically. For example, they would rarely overlap with predators that appeared once every three or five years.

However, if the cicadas' life cycle happened to be a multiple of three or five, such as 15, then they would be at risk of facing those three-year or five-year predators whenever they emerged. In other words, the special nature of prime numbers can allow periodical cicadas to minimize how often they meet with certain predators.

Having life cycles of different prime numbers also ensures that these enormous cicada populations will only overlap with each other on very rare occasions. In the case of the 13-year and 17-year cicadas, they will only emerge together once every 221 years. If it were not for their different life cycles, they would have to compete with each other for food.

### Keeping Your Money and Data Safe

Prime numbers are also useful to humans, who have used their special qualities to develop various technologies. For example, did you know that prime numbers play a role in making online shopping safe? This is thanks to one of their interesting qualities. Although it is relatively simple to multiply one large prime number by another, it is extremely difficult to figure out those numbers when you only have their product\*. In fact, it can take many years to find those prime numbers, even for a powerful computer! Internet security systems use this feature of prime numbers in order to keep your information safe.

Whenever you purchase something online, a very large number,  $X$ , is sent to your computer. Your computer uses this number to turn your credit card number into a code, which it sends back to the website. The keys to decoding this information are  $X$ 's prime factors, the prime numbers that produce  $X$  when they are multiplied together. Even if someone stole the coded information and knew the value of  $X$ , it would be nearly impossible for them to determine the number's prime factors and decode the information. If it were not for this special feature of prime numbers, thieves would find it much easier to access your money and information. These days, mathematicians are still working hard to discover more prime numbers. In addition, there is even an organization that offers a huge award to whoever can find new ones. So far, the largest one known, which was discovered in 2018, is 24,862,048 digits long! Thanks to their usefulness and mysterious nature, it's no wonder that people find prime numbers so fascinating.

## Reading 2

**Game Theory in Decision-Making**

Imagine that the police are investigating you and your friend for a crime. They are questioning you both in different rooms, so you can't communicate with each other. You refuse to answer any questions. Eventually, the police tell you that they will make an offer to both you and your friend: If both of you choose to remain silent, then you will each have to go to prison for one year. However, if one of you confesses and the other stays silent, then whoever confesses will be set free, while the other will go to prison for twenty years. If both of you confess, then you will each go to prison for five years. If you were in this situation, what would you do?

The situation presented above is called the prisoner's dilemma, and it is an example of the type of situation that is analyzed in "game theory." Game theory is a mathematical approach to the study of human interaction and behavior. More specifically, it analyzes situations where individuals, or "players," make interdependent decisions. In such situations, or "games," each individual has to consider how others might act when figuring out what to do. Game theory approaches such situations mathematically in order to determine the most optimal decisions that the "players" can make.

What was your decision in the imaginary situation above? Game theory shows that the most rational strategy for each player is to confess. That is because confessing brings about a better outcome for the individual no matter what the other player decides to do. The most rational strategy for the individual, though, is not always the best strategy for the group as a whole. The ideal outcome for the group, in which the total amount of time spent in prison is only two years, can only be achieved if both players remain silent. Do you think you could trust your friend enough to make such a risky decision?

Many real-world situations show that strategies based on self-interest can lead to bad outcomes for the individual as well as the group. Consider, for instance, two competing soft drink companies. Company A wants to increase sales, so they strategically decide to lower the price of their drinks, as this will draw customers away from Company B. In this situation, Company B will have to do the same in order to stay competitive. In the end, the lower prices will reduce profits for both. Despite being competitors, both companies would have been better off maintaining a stable price.

Game theory studies situations like the prisoner's dilemma so that we can be better prepared to make decisions in the real world. Moreover, by showing us how the choices we make are connected with those of others, it leads us to a deeper understanding of human social interaction. In this way, it can benefit society by promoting cooperative behavior that brings about optimal outcomes for everyone.